ERRATA FOR
INTRODUCTION TO SYMPLECTIC TOPOLOGY
THIRD EDITION

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Abstract. These notes correct a few typos and errors in *Introduction to Symplectic Topology* (3rd edition, Oxford University Press 2017). We thank Katrin Wehrheim, Chris Wendl, and Fabian Ziltener for pointing out errors.

p 100: The factor in equation (3.1.4) should be -1 instead of $1/2$. The correct formula is

$$\{F,\{G,H\}\}+\{G,\{H,F\}\}+\{H,\{F,G\}\}=d\tau(X_F,X_G,X_H).$$

p 109, Lemma 3.2.1: The **Moser Isotopy Lemma** can be strengthened.

Let (M,ω) be a $2n$-dimensional smooth manifold, let $Q\subset M$ be a closed submanifold, and let ω_0 and ω_1 be two symplectic forms on M that agree on T_QM. Then there exist open neighbourhoods $\mathcal{N}_0$ and $\mathcal{N}_1$ of Q and a diffeomorphism $\psi:\mathcal{N}_0\to\mathcal{N}_1$ of Q such that $\psi^*\omega_1=\omega_0$ and

$$(1)\qquad q\in Q,\quad v\in T_qM \qquad\Longrightarrow\qquad \psi(q)=q \quad\text{and}\quad d\psi(q)v=v.$$

The proof does not change. The key observation is that an isotopy ψ_t satisfies (1) if and only if it is generated by a family of smooth vector fields X_t on M that satisfy

$$(2)\qquad X_t|_Q=0,\qquad [X_t,Y]|_Q=0$$

for all t and every vector field Y on M. For the 1-form σ in the proof of Lemma 3.2.1 this translates into the condition that, for every vector field Y on M, the function

$$f_Y:=\iota(Y)\sigma:M\to\mathbb{R}$$

vanishes to first order along Q, i.e.

$$(3)\qquad q\in Q,\quad v\in T_qM \qquad\Longrightarrow\qquad f_Y(q)=0\ \text{ and }\ df_Y(q)v=0.$$

The 1-form σ on a neighbourhood of Q, defined on page 110, satisfies (3) because $\partial_t\phi_t(q)=0$ and $\tau(q;v,w)=0$ for all $q\in Q$ and all $v,w\in T_qM$.

p 114, line -16: At the end of the proof of Step 3 it should be mentioned that one must use Step 1 to obtain a Hamiltonian isotopy $\{\phi_t\}_{0\le t\le 1}$ of M that satisfies $\phi_0=\mathrm{id}$ and $\phi_t\circ\Psi_0\circ\chi_{1,0}=\Psi_t\circ\chi_{1,t}$ for all t, and that this Hamiltonian isotopy satisfies the requirements of part (ii) of Theorem 3.3.1.

p 114, line -7: The term "$\Psi_t^*\omega\in\Omega^2(M)$" should read "$\Psi_t^*\omega\in\Omega^2(\mathbb{R}^{2n})$".

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p 120, Theorem 3.4.10: The proof of the **Symplectic Neighbourhood Theorem** requires the strengthened form of the **Moser Isotopy Lemma** mentioned above (see page 109).

p 147, line -19: The sentence should read *"Examples by Eliashberg* [184] *show that weak and strong fillability differ in dimension* 3 *and Massot–Niederkrüger–Wendl* [440] *proved that they differ in dimension* 5*. The question of weak versus strong fillability is open in dimensions* 7 *and higher."*

p 147, line -17: The sentence should be expanded as follows:
"By a result of Eliashberg [178] *and Gromov* [287] *overtwisted contact 3-manifolds are never weakly fillable. A similar result holds in higher dimensions by results of Niederkrüger* [N]*, Massot–Niederkrüger–Wendl* [440]*, and Borman–Eliashberg–Murphy* [75]*. The heart of the proof is a result by Niederkrüger* [N] *which asserts that a contact manifold containing a 'plastikstufe' is not strongly fillable. In Massot–Niederkrüger–Wendl* [440] *it is explained how the same argument shows that a 'small plastikstufe' obstructs weak fillability, and the existence of a 'small plastikstufe' is an easy consequence of Borman–Eliashberg–Murphy flexibility."*

p 147, last paragraph: There are some inaccuracies in the discussion of the literature. The paragraph should be rewritten as follows.
"An elementary 2-dimensional argument shows that a Liouville domain can have a disconnected (convex) boundary (Example 3.5.29 and Definition 3.5.32). That this phenomenon also occurs in higher dimensions was shown by McDuff [451] and Mitsumatsu [M] *in dimension four and by Geiges [260] in dimensions four and six. Thus fillable contact manifolds do not have to be connected. Examples in all dimensions appear in the work of Massot–Niederkrueger–Wendl [440], where they are an essential ingredient in their construction of nonfillable tight contact manifolds. Using fillable disconnected contact* 3*-manifolds, Albers–Bramham–Wendl [19] constructed examples (attributed to Etnyre) of nonseparating contact hypersurfaces in certain closed 4-dimensional symplectic manifolds. However not all contact manifolds support such an embedding, and also there are restrictions on the ambient symplectic manifold."*

p 425: The path $\beta : [0,1] \to [0,1]$ in Exercise 11.1.11 is required to satisfy the condition $\beta(0) = 0$.

p 531: The first sentence in part (ii) of Remark 13.3.28 should read: *"In* [431]*, Liu also proved that a minimal closed symplectic four-manifold* (M, ω) *is rational or ruled if and only if the symplectic form* ω *is homotopic to a symplectic form* ω' *that satisfies* $K \cdot [\omega'] < 0$*."* (Ruled surfaces over curves of genus at least two admit symplectic forms ω that satisfy $K \cdot [\omega] \geq 0$.)

References

[N] Klaus Niederkrüger, The plastikstufe - a generalization of the overtwisted disk to higher dimensions. *Algebraic & Geometric Topology* **10** (2006), 2385–2429.

[M] Yoshihiko Mitsumatsu, Anosov flows and non-Stein symplectic manifolds. *Annales de l'Institut Fourier* **45** (1995), 1407–1421.

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